Optional Summer Enrichment for Rising IB Math Analysis Higher Level Year 2
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Resources: IB Math AA Guide , Revision Village , Mr. Flynn IB

Learning target 1: I will read the guidelines and reminders for the Math IA, so that I understand expectations. I'll know I've got it when I can confidently communicate about the rubric. chart

Learning target 2: I will read sample IAs, so that I can brainstorm my aim. I'll know I've got it when I can propose 3 potential aims and the relevant math. cookies, popcorn, chess, rowing

Learning target 3: I will complete the IB practice problems, so that I can review and apply prior math learning from Algebra 1 through Analysis 1. I'll know I've got it when I check my answers.

The IB Math Internal Assessment is a **written math exploration**, intended for a peer audience. The paper should be logically developed and easy to follow. Write complete sentences, showcase equations neatly on separate lines, be very detailed when labeling graphs, tables, and diagrams. Please **use the rubric** as a guide to help you write your paper! Provide evidence of reflection throughout the paper. Show your personal engagement through your design, work, and actions. Personal engagement is NOT a section heading in your paper.

- ☐ Through the IB math IA, students will develop math insight, ask questions about math, complete work over an extended period of time, apply math independently, experience the beauty and power of math, use and appreciate technology, develop patience and perseverance, reflect on the significance of their work, and demonstrate math growth.

- ☐ The IB math IA counts as 20% of the overall IB math grade and is worth approximately one test grade for the class grade. The IB math IA is awarded a grade out of 20 based on a rubric with **5 criteria**:
  - ☐ presentation (4) - coherence, organization, & conciseness,
  - ☐ math communication (4) - math language, diagrams/graphs, symbols, & labels,
  - ☐ personal engagement (3) - own it, your initiative and voice drive the exploration,
  - ☐ reflection (3) - discussion is meaningful, critical, insightful, & present throughout
  - ☐ use of mathematics (6) - math is relevant, commensurate, & correct throughout

- ☐ Former students shared: the most challenging task, which can also be the most exciting, is to determine the aim of your paper. Literally: **what is the goal of your paper?** Once you determine the goal, you will determine the appropriate and relevant math (from the IB Course Guide) needed to pursue your goal. As you investigate, you may decide to "tweak" your aim, in an effort to apply and "show off" your relevant math skills. The paper will consist of your writing, math representations (graphs), and math calculations.

☐ The paper must have a **cover page** with the

a) **title** of the exploration. The title should indicate where the stimulus has taken the student. Instead of "number patterns", try "number patterns - exploring patterns in final digits of prime numbers.
b) **page count**. The final paper should be **12-20 pages, double spaced.** Part of the rubric assesses the "conciseness" of the work. Think "quality over quantity". Some of the best IAs are 12 pages.
c) **Do not include your name** or any personal information anywhere in the paper.

☐ The paper must have a **bibliography**

a) Select and follow a method for citing sources, like MLA or APA. Be consistent.
b) While you research and collect data, keep a record of websites and other sources you use.

☐ Structure and **sections headings** - there is no requirement, and each paper is unique! Headings may be used to enhance the organization. Possible section headings:

I. Introduction and Rationale

a) State your research question
b) **Clearly state the aim of the paper** - this is the goal or the purpose - everything must tie back to the aim
c) Why did you choose this topic? Rationale is no longer a required element, but explaining what led you to your topic can help develop your purpose and goal.
d) Ask and answer personal questions, like "I wonder if….",
e) Make conjectures and then design a plan to test them out

II. Aim and Approach

a) **Focus on the aim**, avoid irrelevance. The aim should drive your process and math
b) Avoid being repetitive.
c) State appropriate definitions and explanations of concepts.
d) Define vocabulary terms that may be unfamiliar to readers.
e) **Define variables**, use appropriate scripts and fonts.
f) Use technology to create diagrams, tables, graphs, etc, that are clear and labeled.
g) Use precise notation, relevant graphs (with labels!), and math computations.
h) The reader should be able to read the paper without pausing, or having to re-read, or flip pages to see an image. Presentation is part of the rubric.
i) Include graphs, tables, diagrams, and computations in appropriate places.
j) Tables may NOT exceed one page. You **cannot break a table** over multiple pages. You may need to use creative formatting to make a table fit on one page.
k) Use the **DESMOS graphing calculator** to create and import beautiful graphs.

III. Data Collection and Results (Calculations)

a) If you use decimals, consider and discuss the degree of accuracy you used. How many decimal places are relevant in your exploration and why? Consider, for example, how you might round when measuring milligrams of water in a pool versus milligrams of medication administered. 3 sig figs is the norm for IB math.

b) Use proper math symbols and fonts.

c) Use the equal sign (=) for exact values, use the approximately equal sign (≈) for rounded values. IB is VERY picky about this!

d) Use **proper math vocabulary**. For example, say "substitute", do not say "plug in"

e) Do **NOT use calculator notation, like * and ^**

f) Include graphs, tables, diagrams, and computations in appropriate places.

g) Tables may NOT exceed one page. You may need to use creative formatting.

h) Use the DESMOS graphing calculator to create and import beautiful graphs.

i) **Make sure you understand the math that you use.** If you do not understand the math, it will be obvious to the reader. Part of the rubric assesses your ability to demonstrate your knowledge and understanding of the math you used.

IV. Analysis and Conclusion

a) Tie up all the major ideas of your paper.

b) Answer the question, "So what?" What is the significance of this work?

c) Answer the question, "How useful are my findings?"

d) If I studied this further, where would I take it and why?

e) What went wrong, and why?

f) Are my conclusions and model reasonable?

g) Think and reflect along the lines of TOK.

h) Consider historical perspectives - what has happened with your topic in the past?

i) Consider global perspectives - make links between your own life and lives of others throughout the world.

j) Discuss the implications of your results. What do they mean? What are they important? How do they affect your life?

k) Discuss your results in the context of your topic, not just in general terms. For example, instead of saying "the graph levels off at $x > 15$", in context you might say, "the graph levels off after the age of 15 because that is the average age when girls tend to reach their maximum height".

l) Discuss possible limitations of your topic, like a restriction or a defect or a failing

m) Discuss possible extensions of your topic

n) Make connections between your topic and different disciplines

o) Add your voice throughout your paper. This work should be individualized and unique to you, not something that could be found in a textbook explanation.

p) You need to personally engage in your topic. For example, if you were to study the performance of the mile runners on the track team, you would collect their

times, analyze the data statistically and exhaustively (using every math technique you have learned thus far). Then, draw some conclusions about their performance and how it could improve. Think of how you could analyze the runners' performance using all the math knowledge you have accumulated thus far. In your conclusion, you should discuss the results, but more importantly, how *reasonable* your results and conclusions appear. Also think about how to apply what you learned or if there were any improvements which could be made to your study. You can also comment on what you learned through this investigation.

☐ **Bibliography**

a) Select and follow a method for citing sources, like MLA or APA. Be consistent.
b) While you work, keep a record of websites and other sources you use.
c) The bibliography does NOT count toward the 12-20 page guideline.

☐ **Appendix** - an appendix may be used for data/tables that exceed a full page and/or for other information that is not important in the body of the paper, but may be helpful for reference. An appendix is **NOT** required. A table of contents is **NOT** recommended.

➢ **Criterion A: PRESENTATION (4)** - This criterion assesses the organization and coherence of the exploration. Goals: coherent, well-organized, concise.

- A <u>coherent</u> exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.
- A <u>well-organized</u> exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.
- A <u>concise</u> exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.
- The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

➔ Student expresses ideas clearly and logically
➔ Student identifies a clear aim, focuses on the aim, and avoids irrelevance
➔ Student structures ideas in a logical manner
➔ Student edits the exploration so that it is easy to follow
➔ Student cites references where appropriate
➔ Student includes an introduction, explanation of aim, body, conclusion.
➔ Everything links back to the aim.
➔ Student includes relevant graphs, tables, and diagrams in an appropriate place

0 The exploration does not reach the standard described by the descriptors below.

1 The exploration has some coherence or some organization.

2 The exploration has some coherence and shows some organization.

3 The exploration is coherent and well organized.

4 The exploration is coherent, well organized, and concise.

➢ **Criterion B: MATHEMATICAL COMMUNICATION (4)** – math language, math symbols, diagrams/graphs and labels. This criterion assesses to what extent the student has:

- used appropriate mathematical language (notation, symbols, terminology). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work.
- defined key terms and variables, where required
- used multiple forms of mathematical representation, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a deductive method and set out proofs logically where appropriate
- Examples of level 1 can include graphs not being labelled, consistent use of calculator notation with no other forms of correct mathematical communication.
- Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any minor errors that do not impair clear communication should not be penalized.

➔ Student uses appropriate mathematical language and representation (notation, symbols, terminology)
➔ Student defines key terms, where required
➔ Student expresses results to an appropriate degree of accuracy
➔ Student uses multiple forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
➔ Student labels all diagrams, graphs, and table (title, axes, scale).
➔ Student uses a deductive method and sets out proofs logically where appropriate
➔ Student selects and uses appropriate mathematical and technology tools to enhance mathematical communication (GDC, screenshots, graphing, spreadsheets, databases, drawing and word-processing software)

0 The exploration does not reach the standard described by the descriptors below.

1 The exploration contains some relevant mathematical communication which is partially appropriate.

2 The exploration contains some relevant mathematical communication.

3 The mathematical communication is relevant, appropriate and mostly consistent.

4 The mathematical communication is relevant, appropriate and consistent throughout.

➢ **Criterion C: PERSONAL ENGAGEMENT (3)** – This criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort. Goal: the student's unique voice drives exploration.

- Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. There must be evidence of personal engagement demonstrated in the student's work. Textbook style explorations or reproduction of readily available mathematics without the candidate's own perspective are unlikely to achieve the higher levels.
- Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer's intentions.
- Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer's intentions.

➔ Student asks questions (what if…?, why…?), makes conjectures, tests predictions, investigates mathematical ideas
➔ Student addresses interest, reads mathematics and researches areas of interest
➔ Student looks for and creates mathematical models for real-world situations
➔ Student considers historical and global perspectives. Student explores math from "different" perspectives.
➔ Student demonstrates that the student has "made the exploration his/her own"
➔ Student expresses ideas in an individual way. Student designs own surveys and/or collects own data.
➔ Student thinks independently and/or creatively & presents math ideas in their own way
➔ Student explores unfamiliar mathematics.

0 The exploration does not reach the standard described by the descriptors below.

1 There is evidence of some personal engagement. *Examples: Textbook style explorations. Reproduction of readily available mathematics without the candidate's own perspective*

2 There is evidence of significant personal engagement. (evident on a few occasions, drive the exploration forward)

3 There is evidence of outstanding personal engagement. (evident on numerous occasions and high quality)

➢ **Criterion D: REFLECTION (3)** – This criterion assesses how the student reviews, analyses and evaluates the exploration. Goals: meaningful, critical, insightful, present throughout.

- Although reflection may be seen in the conclusion to the exploration, it may also be

found throughout the exploration. Simply describing results represents limited reflection. Further consideration is required to achieve the higher levels.

- Some ways of showing meaningful reflection are: linking to the aims of the exploration, commenting on what they have learned, considering some limitations or comparing different mathematical approaches.
- Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.
- Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

➔ The student reviews, analyzes and evaluates the exploration throughout the work

➔ Student discusses the implications of results and the impact on student's understanding of the topic. "What next?"

➔ Student discusses the strengths and weaknesses of approaches; comparing different mathematical approaches

➔ Student considers different perspectives

➔ Student discusses possible limitations and/or extensions

➔ Student makes links to different fields and/or areas of mathematics

0 The exploration does not reach the standard described by the descriptors below.

1 There is evidence of limited reflection. (simply describes results)

2 There is evidence of meaningful reflection.
(links to aims, what they've learned, some limitations/extensions)

3 There is substantial evidence of critical reflection. (deeply insightful, deciding in a way that develops the paper). *If reflection only appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.*

➢ **Criterion E: USE OF MATHEMATICS SL (6)** - Goals: relevant, commensurate, correct throughout. This SL criterion assesses to what extent students use mathematics that is relevant to the exploration.

- "Relevant" refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.
- Students are expected to produce work that is commensurate with the level of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, or at a similar level.
- Demonstrate means "to make clear by reasoning or evidence, illustrating with examples or practical application". Obtaining the correct answer is not sufficient to demonstrate understanding or "some understanding" in order to achieve level 2 or higher.
- For knowledge and understanding to be thorough it must be demonstrated

throughout. The mathematics can be regarded as correct even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

- Students are encouraged to use technology to obtain results where appropriate, but understanding must be demonstrated. In order for the student to achieve higher than level 1, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.
- The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with AA, and well-explained by the student, then it can achieve 3/6.

➔ Student demonstrates knowledge/understanding by reasoning, evidence, illustrating with examples, practical application

➔ Student uses relevant math to support the development of the exploration towards the completion of its aim

➔ Student applies mathematics in different contexts

➔ Student applies problem-solving techniques

➔ Student recognizes and explains patterns, looks at problem from different perspectives, links to different areas of math

➔ Correct and precise: Precise mathematics is error-free and uses an appropriate level of accuracy at all times

➔ Commensurate: mathematics used is from the IB Math Analysis & Approaches Guide (not solely prior learning).

➔ Student uses technology where appropriate while also demonstrating understanding

➔ Student makes conclusions clear with reasoning and evidence, illustrating with examples or practical application.

➔ Lines of reasoning must be shown to justify steps in developing the exploration; math claims must be justified or proven.

0 The exploration does not reach the standard described by the descriptors below.

1 Some relevant mathematics is used. Limited understanding is demonstrated.
Example: Merely substituting values into a formula. This does not necessarily demonstrate understanding of the results. Just obtaining the correct answer without reasoning or evidence.

2 Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.

3 Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated

4 Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge & understanding are demonstrated.

5 Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication and rigor. Thorough knowledge & understanding are demonstrated.

6 Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigor. Thorough knowledge & understanding are demonstrated. Sophisticated = HL or SL used complexly.